Op-Amp Circuit Analysis by Inspection

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The transfer function of a linear system can be represented in the general form

$$H(s) = H_0 \frac{(1 + s/\omega_{z1})(1 + s/\omega_{z2}) \cdots (1 + s/\omega_{z,m})}{(1 + s/\omega_{p1})(1 + s/\omega_{p2}) \cdots (1 + s/\omega_{p,n})} \tag{1}$$

where the poles $\omega_{p1},\ \omega_{p2},\ \ldots,\ \omega_{p,n}$ and zeros $\omega_{z1},\ \omega_{z2},\ \ldots,\ \omega_{z,m}$ are roots of the denominator and numerator, the values for which the function goes to infinity and zero, respectively.

1 General Method

1. To find the poles, set the input (v_{in}) to zero, or to find the zeros, set the output (v_{out}) to zero.
2. Find the equivalent resistance, R_{eq}, seen by each independent capacitor, C_{ind}, in the input and feedback networks.
   - Capacitors in series ($C_1 \parallel C_2$) or in parallel ($C_1 + C_2$) are combined to form a single independent capacitor.
3. Calculate the pole and zero frequencies as $\omega_x = 1/\left(R_{eq}C_{ind}\right)$.

2 Approximate Method

1. Series RC pairs in the input network form a pole, and pairs in the feedback network form a zero.
2. Parallel RC pairs in the input network form a zero, and pairs in the feedback network form a pole.
3. Calculate the pole and zero frequencies as $\omega_x = 1/(RC)$.

3 Examples

3.1 LEAD COMPENSATOR

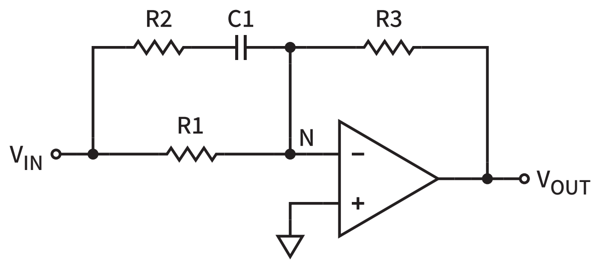


Fig. 1. Lead compensator schematic.

The DC gain for the inverting configuration is

$$H_0 = -R_3/R_1 \tag{2}$$

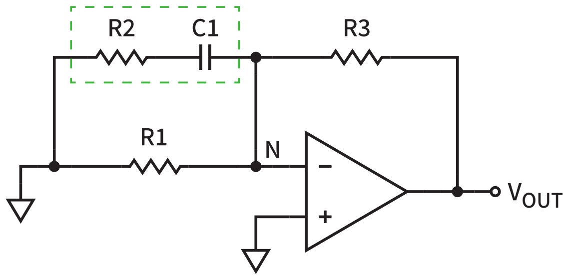
by considering C_1 as an open circuit (Fig. 1).

Obtain the poles by zeroing the input ($v_{\text{in}} = 0$) and finding the equivalent resistance seen by C_1 (Fig. 2(a)). Note that node N, the op-amp's negative input terminal, is a virtual ground.

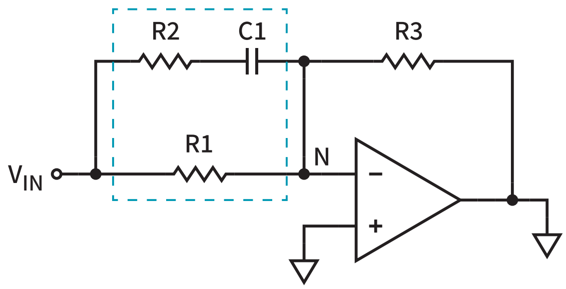
$$\omega_{\text{p}} = 1/(R_2 C_1) \tag{3}$$

Likewise, obtain the zeros by zeroing the output ($v_{\text{out}} = 0$) (Fig. 2(b)).

$$\begin{aligned} \omega_{\text{z}} &= 1/\left[(R_1 + R_2)\, C_1\right] \\ &\approx 1/(R_1 C_1) \quad \text{for } R_1 \gg R_2 \end{aligned} \tag{4}$$



(a)



(b)

Fig. 2. Lead compensator equivalent circuit for finding (a) poles and (b) zeros.

The complete transfer function can be put in the form (Fig. 3)

$$
\begin{aligned}
H(s) &= H_0 \frac{1 + s/\omega_\mathrm{z}}{1 + s/\omega_\mathrm{p}} \\
&= -\frac{R_3}{R_1} \cdot \frac{1 + s(R_1 + R_2)C_1}{1 + sR_2C_1} \\
&\approx -\frac{R_3}{R_1} \cdot \frac{1 + sR_1C_1}{1 + sR_2C_1} \quad \text{for } R_1 \gg R_2\,.
\end{aligned}
\tag{5}
$$

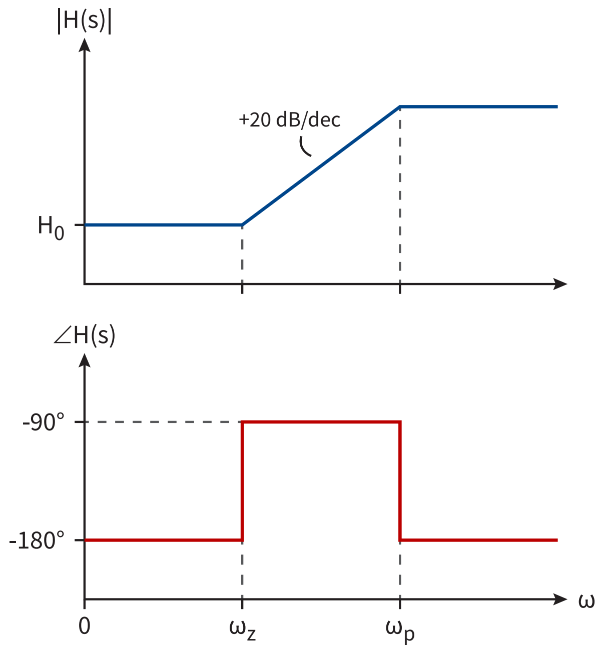


Fig. 3. Lead compensator Bode plot.

3.2 Type 2 Compensator

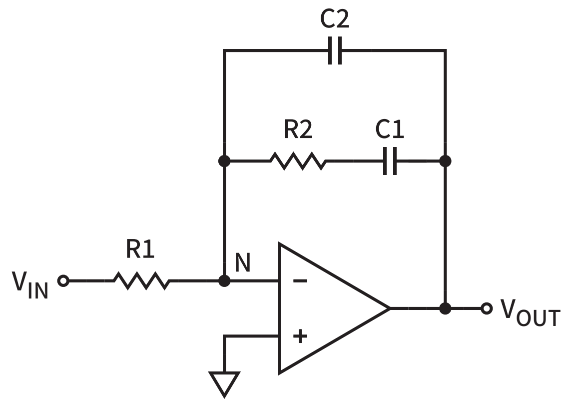


Fig. 4. Type 2 compensator schematic.

The DC gain formula for an inverting configuration is not valid here because the feedback impedance at DC is undefined. Instead, the constant value $H_0 = -1/(RC)$ in the integrator transfer function represents the gain at $\omega = 1\,\text{rad/s}$ and is determined by the input resistance and feedback capacitance (Fig. 4).

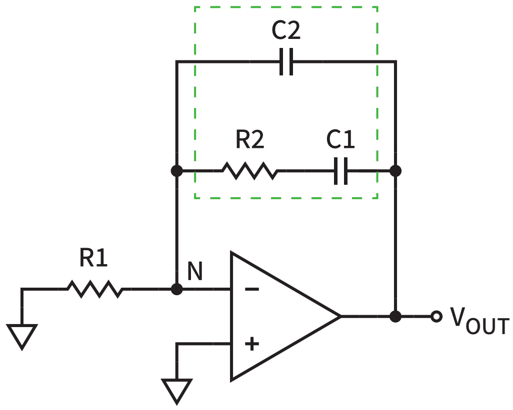
$$\begin{aligned} H_0 &= -1/\left[R_1\left(C_1 + C_2\right)\right] \\ &\approx -1/\left(R_1 C_1\right) \quad \text{for } C_1 \gg C_2 \end{aligned} \tag{6}$$

Obtain the poles by zeroing the input ($v_{\text{in}} = 0$) and finding the equivalent resistance seen by C_1 and C_2 (Fig. 5(a)). Note that node N, the op-amp's negative input terminal, is a virtual ground.

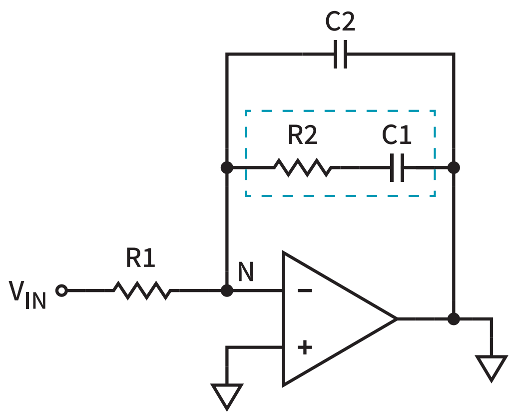
$$\begin{aligned} \omega_{\text{p}} &= 1/\left[R_2\left(C_1 \parallel C_2\right)\right] \\ &\approx 1/\left(R_2 C_2\right) \quad \text{for } C_1 \gg C_2 \end{aligned} \tag{7}$$

Likewise, obtain the zeros by zeroing the output ($v_{\text{out}} = 0$) (Fig. 5(b)).

$$\omega_{\text{z}} = 1/\left(R_2 C_1\right) \tag{8}$$



(a)



(b)

Fig. 5. Type 2 compensator equivalent circuit for finding (a) poles and (b) zeros.

The complete transfer function can be put in the form (Fig. 6)

$$\begin{aligned} H(s) &= H_0 \frac{1 + s/\omega_z}{s\left(1 + s/\omega_p\right)} \\ &= -\frac{1}{R_1\left(C_1 + C_2\right)} \cdot \frac{1 + sR_2C_1}{s\left[1 + sR_2\left(C_1 \parallel C_2\right)\right]} \\ &\approx -\frac{1}{R_1C_1} \cdot \frac{1 + sR_2C_1}{s\left(1 + sR_2C_2\right)} \quad \text{for } C_1 \gg C_2\,. \end{aligned} \tag{9}$$

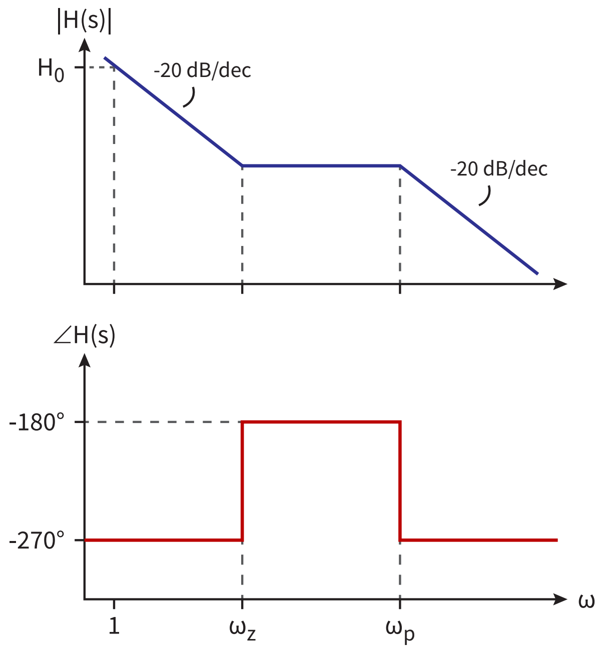


Fig. 6. Type 2 compensator Bode plot.

3.3 TYPE 3 COMPENSATOR

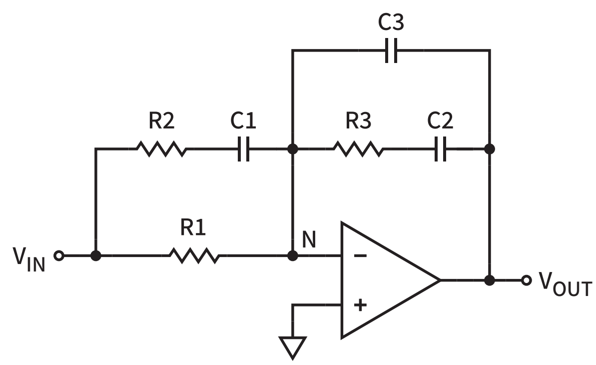


Fig. 7. Type 3 compensator schematic.

The DC gain formula for an inverting configuration is not valid here because the feedback impedance at DC is undefined. Instead, the constant value $H_0 = -1/(RC)$ in the integrator transfer function represents the gain at $\omega = 1\,\text{rad/s}$ and is determined by the input resistance and feedback capacitance (Fig. 7).

$$\begin{aligned} H_0 &= -1/\left[R_1\left(C_2 + C_3\right)\right] \\ &\approx -1/\left(R_1 C_2\right) \quad \text{for } C_2 \gg C_3 \end{aligned} \tag{10}$$

Obtain the poles by zeroing the input ($v_{in} = 0$) and finding the equivalent resistances seen by C_1, C_2, and C_3 (Fig. 8(a)). Note that node N, the op-amp's negative input terminal, is a virtual ground.

$$\omega_{p1} = 1/\left(R_2 C_1\right) \tag{11}$$

$$\begin{aligned} \omega_{p2} &= 1/\left[R_3\left(C_2 \parallel C_3\right)\right] \\ &\approx 1/\left(R_3 C_3\right) \quad \text{for } C_2 \gg C_3 \end{aligned} \tag{12}$$

Likewise, obtain the zeros by zeroing the output ($v_{out} = 0$) (Fig. 8(b)).

$$\begin{aligned} \omega_{z1} &= 1/\left[\left(R_1 + R_2\right) C_1\right] \\ &\approx 1/\left(R_1 C_1\right) \quad \text{for } R_1 \gg R_2 \end{aligned} \tag{13}$$

$$\omega_{z2} = 1/\left(R_3 C_2\right) \tag{14}$$

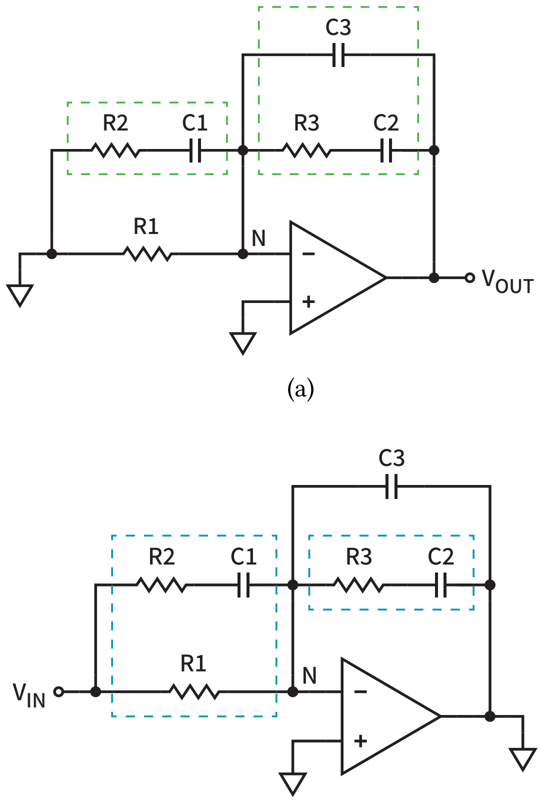


Fig. 8. Type 3 compensator equivalent circuit for finding (a) poles and (b) zeros.

The complete transfer function can be put in the form (Fig. 9)

$$\begin{aligned} H(s) &= H_0 \frac{(1+s/\omega_{z1})(1+s/\omega_{z2})}{s\left(1+s/\omega_{p1}\right)\left(1+s/\omega_{p2}\right)} \\ &= -\frac{1}{R_1(C_2+C_3)} \cdot \frac{[1+s(R_1+R_2)C_1](1+sR_3C_2)}{s(1+sR_2C_1)[1+sR_3(C_2 \parallel C_3)]} \\ &\approx -\frac{1}{R_1C_2} \cdot \frac{(1+sR_1C_1)(1+sR_3C_2)}{s(1+sR_2C_1)(1+sR_3C_3)} \quad \text{for } R_1 \gg R_2 \text{ and } C_2 \gg C_3\,. \end{aligned} \tag{15}$$

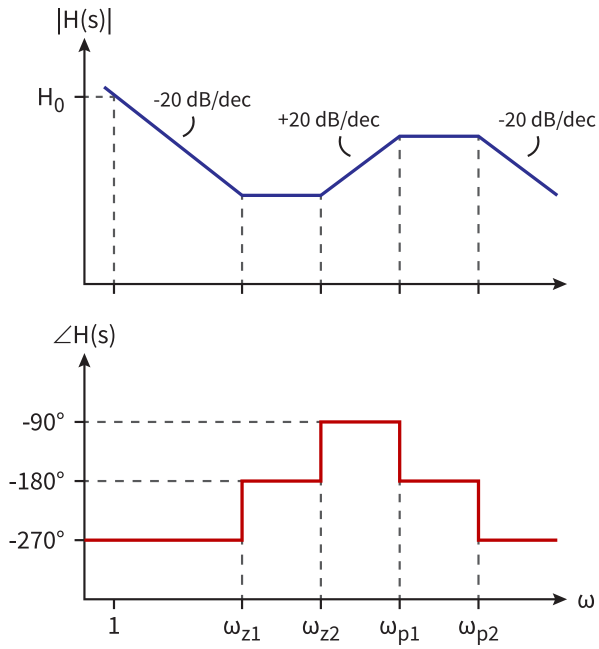


Fig. 9. Type 3 compensator Bode plot.

4 Appendix

4.1 Lead compensator transfer function derivation

$$Z_\mathrm{f} = R_3 \tag{16}$$

$$\begin{aligned} Z_\mathrm{i} &= R_1 \parallel \left(R_2 + \frac{1}{sC_1}\right) \\ &= \frac{R_1\left(R_2 + \frac{1}{sC_1}\right)}{R_1 + R_2 + \frac{1}{sC_1}} \\ &= \frac{R_1 + sR_1R_2C_1}{1 + s(R_1 + R_2)C_1} \end{aligned} \tag{17}$$

$$\begin{aligned} H(s) &= -\frac{Z_\mathrm{f}}{Z_\mathrm{i}} \\ &= -R_3 \cdot \frac{1 + s(R_1 + R_2)C_1}{R_1 + sR_1R_2C_1} \\ &= -\frac{R_3}{R_1} \cdot \frac{1 + s(R_1 + R_2)C_1}{1 + sR_2C_1} \\ &= H_0 \frac{1 + s/\omega_\mathrm{z}}{1 + s/\omega_\mathrm{p}} \end{aligned} \tag{18}$$

$$H_0 = -\frac{R_3}{R_1} \tag{19}$$

$$\omega_\mathrm{p} = \frac{1}{R_2C_1} \tag{20}$$

$$\omega_\mathrm{z} = \frac{1}{(R_1 + R_2)C_1} \tag{21}$$

4.2 Type 2 compensator transfer function derivation

$$\begin{aligned} Z_\mathrm{f} &= \left(R_2 + \frac{1}{sC_1}\right) \parallel \frac{1}{sC_2} \\ &= \frac{\left(R_2 + \frac{1}{sC_1}\right)\frac{1}{sC_2}}{R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}} \\ &= \frac{1 + sR_2C_1}{s\left(C_1 + C_2 + sR_2C_1C_2\right)} \end{aligned} \tag{22}$$

$$Z_\mathrm{i} = R_1 \tag{23}$$

$$\begin{aligned} H(s) &= -\frac{Z_\mathrm{f}}{Z_\mathrm{i}} \\ &= -\frac{1 + sR_2C_1}{s\left(C_1 + C_2 + sR_2C_1C_2\right)} \cdot \frac{1}{R_1} \\ &= -\frac{1}{R_1\left(C_1 + C_2\right)} \cdot \frac{1 + sR_2C_1}{s\left(1 + \frac{sR_2C_1C_2}{C_1+C_2}\right)} \\ &= -\frac{1}{R_1\left(C_1 + C_2\right)} \cdot \frac{1 + sR_2C_1}{s\left[1 + sR_2\left(C_1 \parallel C_2\right)\right]} \\ &= H_0 \frac{1 + s/\omega_\mathrm{z}}{s\left(1 + s/\omega_\mathrm{p}\right)} \end{aligned} \tag{24}$$

$$H_0 = -\frac{1}{R_1\left(C_1 + C_2\right)} \tag{25}$$

$$\omega_\mathrm{p} = \frac{1}{R_2\left(C_1 \parallel C_2\right)} \tag{26}$$

$$\omega_\mathrm{z} = \frac{1}{R_2C_1} \tag{27}$$

4.3 TYPE 3 COMPENSATOR TRANSFER FUNCTION DERIVATION

$$\begin{aligned} Z_\text{f} &= \left(R_3 + \frac{1}{sC_2}\right) \parallel \frac{1}{sC_3} \\ &= \frac{\left(R_3 + \frac{1}{sC_2}\right)\frac{1}{sC_3}}{R_3 + \frac{1}{sC_2} + \frac{1}{sC_3}} \\ &= \frac{1 + sR_3C_2}{s\left(C_2 + C_3 + sR_3C_2C_3\right)} \end{aligned} \tag{28}$$

$$\begin{aligned} Z_\text{i} &= R_1 \parallel \left(R_2 + \frac{1}{sC_1}\right) \\ &= \frac{R_1\left(R_2 + \frac{1}{sC_1}\right)}{R_1 + R_2 + \frac{1}{sC_1}} \\ &= \frac{R_1 + sR_1R_2C_1}{1 + s\left(R_1 + R_2\right)C_1} \end{aligned} \tag{29}$$

$$\begin{aligned} H(s) &= -\frac{Z_\text{f}}{Z_\text{i}} \\ &= -\frac{1 + sR_3C_2}{s\left(C_2 + C_3 + sR_3C_2C_3\right)} \cdot \frac{1 + s\left(R_1 + R_2\right)C_1}{R_1 + sR_1R_2C_1} \\ &= -\frac{1}{R_1\left(C_2 + C_3\right)} \cdot \frac{\left(1 + sR_3C_2\right)\left[1 + s\left(R_1 + R_2\right)C_1\right]}{s\left(1 + \frac{R_3C_2C_3}{C_2+C_3}\right)\left(1 + sR_2C_1\right)} \\ &= -\frac{1}{R_1\left(C_2 + C_3\right)} \cdot \frac{\left[1 + s\left(R_1 + R_2\right)C_1\right]\left(1 + sR_3C_2\right)}{s\left(1 + sR_2C_1\right)\left[1 + sR_3\left(C_2 \parallel C_3\right)\right]} \\ &= H_0\frac{\left(1 + s/\omega_\text{z1}\right)\left(1 + s/\omega_\text{z2}\right)}{s\left(1 + s/\omega_\text{p1}\right)\left(1 + s/\omega_\text{p2}\right)} \end{aligned} \tag{30}$$

$$H_0 = -\frac{1}{R_1 (C_2 + C_3)} \tag{31}$$

$$\omega_{\mathrm{p1}} = \frac{1}{R_2 C_1} \tag{32}$$

$$\omega_{\mathrm{p2}} = \frac{1}{R_3 (C_2 \parallel C_3)} \tag{33}$$

$$\omega_{\mathrm{z1}} = \frac{1}{(R_1 + R_2) C_1} \tag{34}$$

$$\omega_{\mathrm{z2}} = \frac{1}{R_3 C_2} \tag{35}$$